# Decision Trees

Machine Learning
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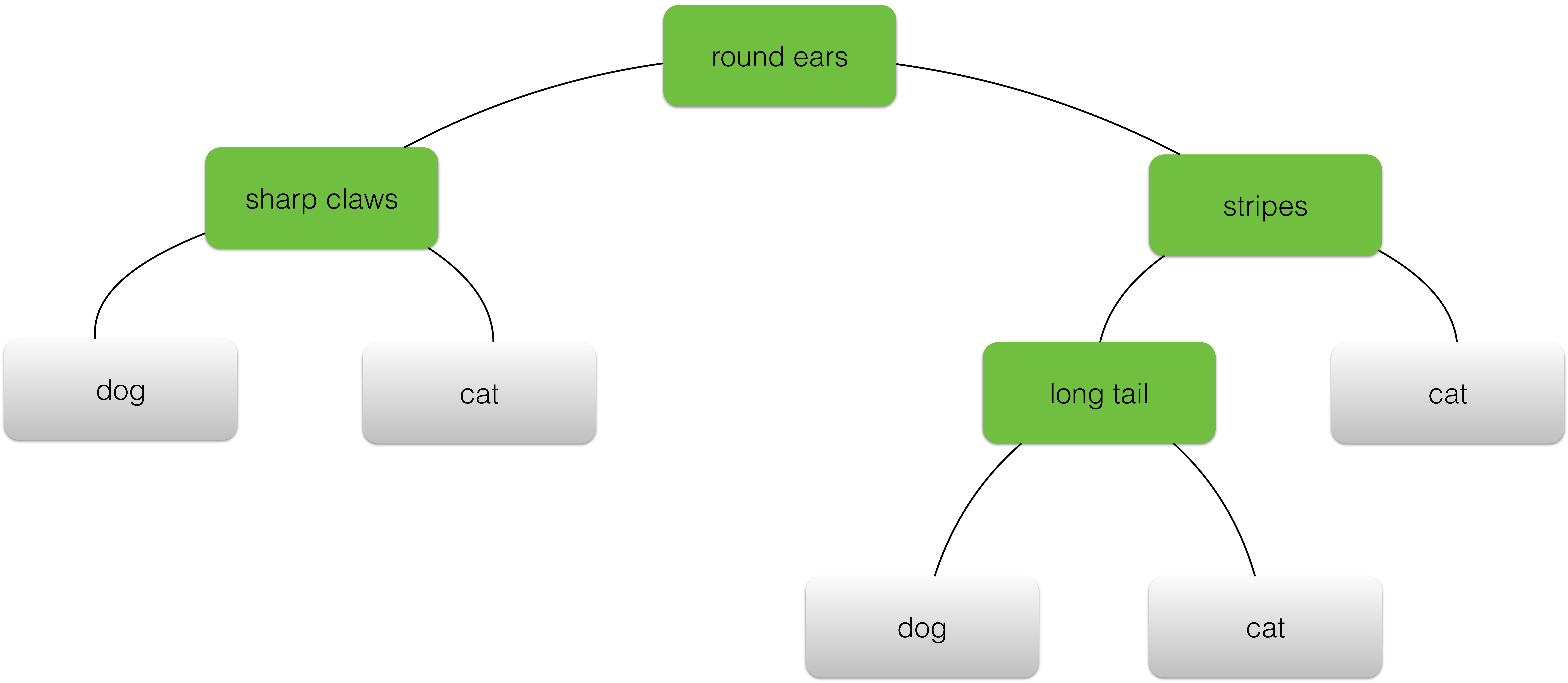
# Outline

- Learning decision trees
- Extensions: random forests


round ears
sharp claws
stripes
dog
cat
long tail
cat
dog
cat

round ears
sharp claws
stripes
dog
cat
long tail
cat
dog
cat

# Decision Tree Learning

- Greedily choose best decision rule
- Recursively train decision tree for each resulting subset

```
function fitTree(D, depth)
  if D is all one class or depth >= maxDepth
    node.prediction = most common class in D
    return node
  rule = BestDecisionRule(D)
  dataLeft = {(x, y) from D where rule(D) is true}
  dataRight = {(x, y) from D where rule(D) is false)}
  node.left = fitTree(D_left, depth+1)
  node.right = fitTree(D_right, depth+1)
```

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function fitTree(D, depth)
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```

# Choosing Decision Rules

- Define a cost function **cost(D)**
  - Misclassification rate
  - Entropy or information gain
  - Gini index

# Misclassification Rate

$$\hat{\pi}_c := \frac{1}{|\mathcal{D}|} \sum_{i \in \mathcal{D}} \mathbb{I}(y_i = c)$$

class proportion
(estimated probability)

$$\hat{y} := \operatorname*{argmax}_c \ \hat{\pi}_c$$

best prediction

$$\mathrm{cost}(\mathcal{D}) := \frac{1}{|\mathcal{D}|} \sum_{i \in \mathcal{D}} \mathbb{I}(y_i \neq \hat{y}) = 1 - \hat{\pi}_{\hat{y}}$$

error rate

$$\mathrm{cost}(\mathcal{D}) - \left( \frac{|\mathcal{D}_L|}{|\mathcal{D}|} \mathrm{cost}(\mathcal{D}_L) + \frac{|\mathcal{D}_R|}{|\mathcal{D}|} \mathrm{cost}(\mathcal{D}_R) \right)$$

cost reduction

# Entropy and Information Gain

$$\hat{\pi}_c := \frac{1}{|\mathcal{D}|} \sum_{i \in \mathcal{D}} \mathbb{I}(y_i = c)$$

$$\mathrm{cost}(\mathcal{D}) - \left( \frac{|\mathcal{D}_L|}{|\mathcal{D}|} \mathrm{cost}(\mathcal{D}_L) + \frac{|\mathcal{D}_R|}{|\mathcal{D}|} \mathrm{cost}(\mathcal{D}_R) \right)$$

$$H(\hat{\boldsymbol{\pi}}) := -\sum_{c=1}^{C} \hat{\pi}_c \log \hat{\pi}_c$$

$$\begin{aligned}
\mathrm{infoGain}(j) &= H(Y) - H(Y|X_j) \\
&= -\sum_{y} \Pr(Y = y) \log \Pr(Y = y) + \\
&\quad \sum_{x_j} \Pr(X_j = x_j) \sum_{y} \Pr(Y = y | X_j = x_j) \log \Pr(Y = y | X_j = x_j).
\end{aligned}$$

# Information Gain

$$\begin{aligned}
\text{infoGain}(j) &= H(Y) - H(Y|X_j) \\
&= -\sum_y \Pr(Y=y)\log\Pr(Y=y) + \\
&\quad \sum_{x_j}\Pr(X_j=x_j)\sum_y \Pr(Y=y|X_j=x_j)\log\Pr(Y=y|X_j=x_j).
\end{aligned}$$

$X_j = Y$

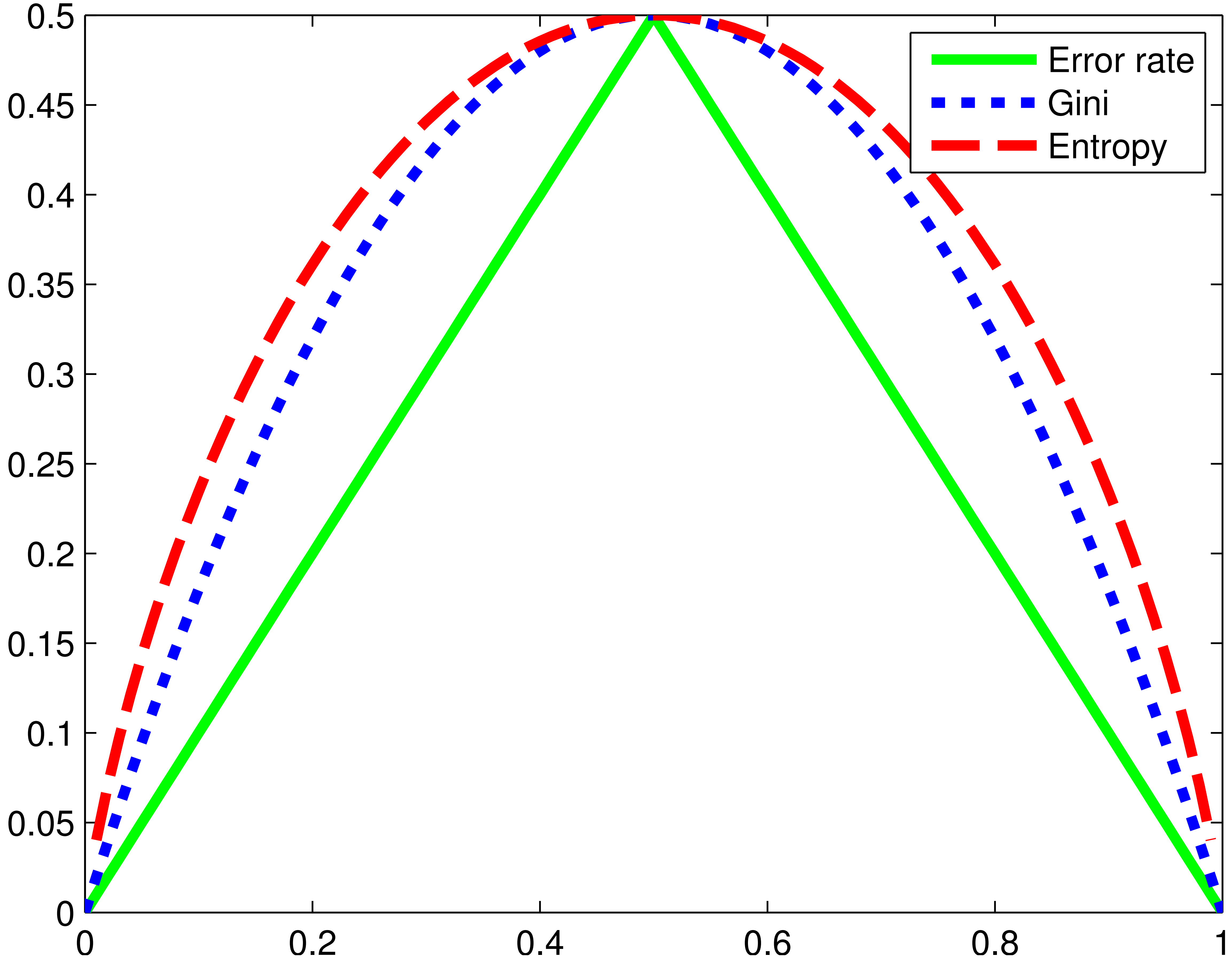
$X_j \perp Y$

# Gini Index

$$\sum_{c=1}^{C} \hat{\pi}_c (1 - \hat{\pi}_c) = \sum_c \hat{\pi}_c - \sum_c \hat{\pi}_c^2 = 1 - \sum_c \hat{\pi}_c^2$$

like misclassification rate, but accounts for uncertainty

# Comparing the Metrics



probability of class 1

```
% Fig 9.3 from Hastie book

p=0:0.01:1;
gini = 2*p.*(1-p);
entropy = -p.*log(p) - (1-p).*log(1-p);
err = 1-max(p,1-p);

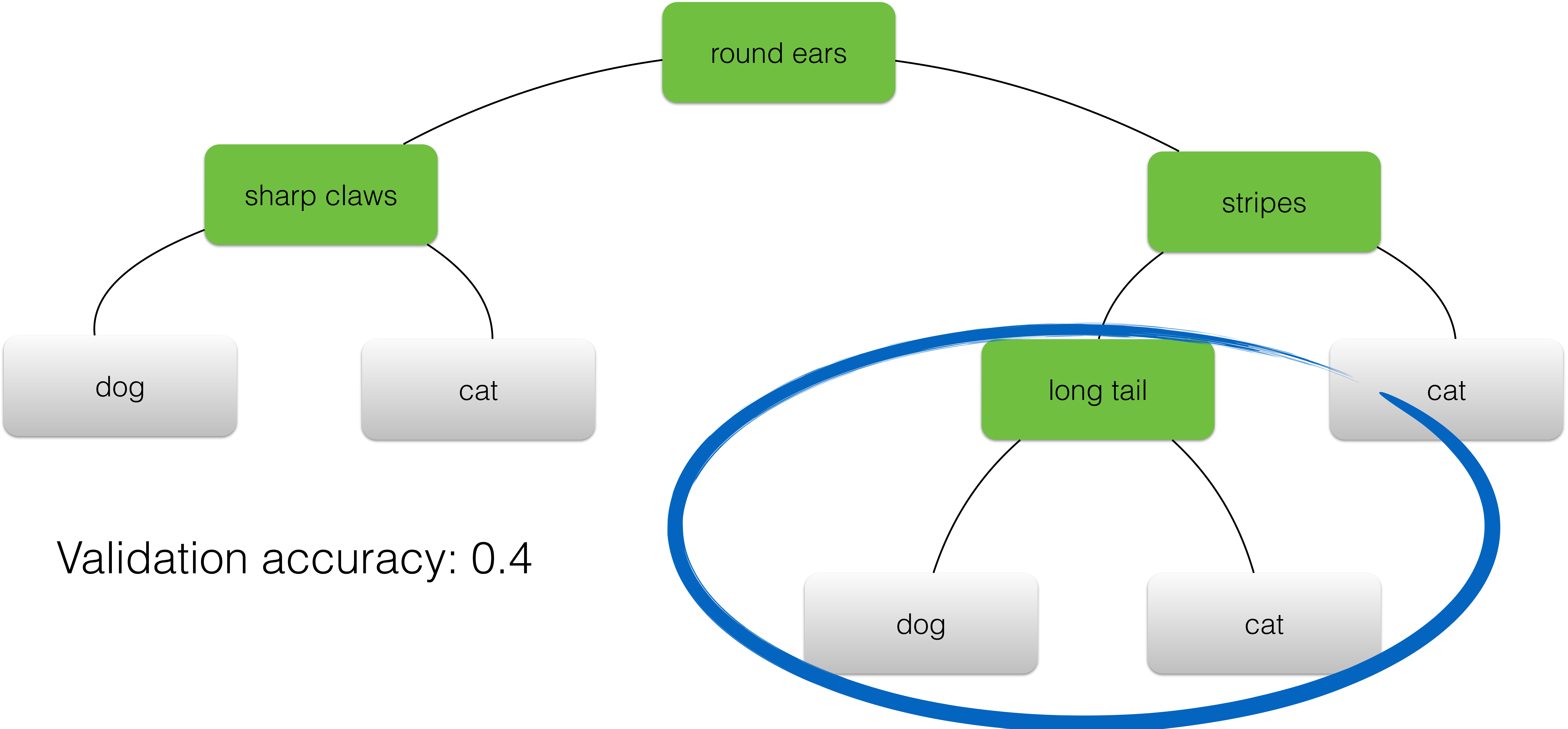
% scale to pass through (0.5, 0.5)
entropy = entropy./max(entropy) * 0.5;

figure;
plot(p, err, 'g-', p, gini, 'b:', p, …
       entropy, 'r--', 'linewidth', 3);
legend('Error rate', 'Gini', 'Entropy')
```

# Overfitting

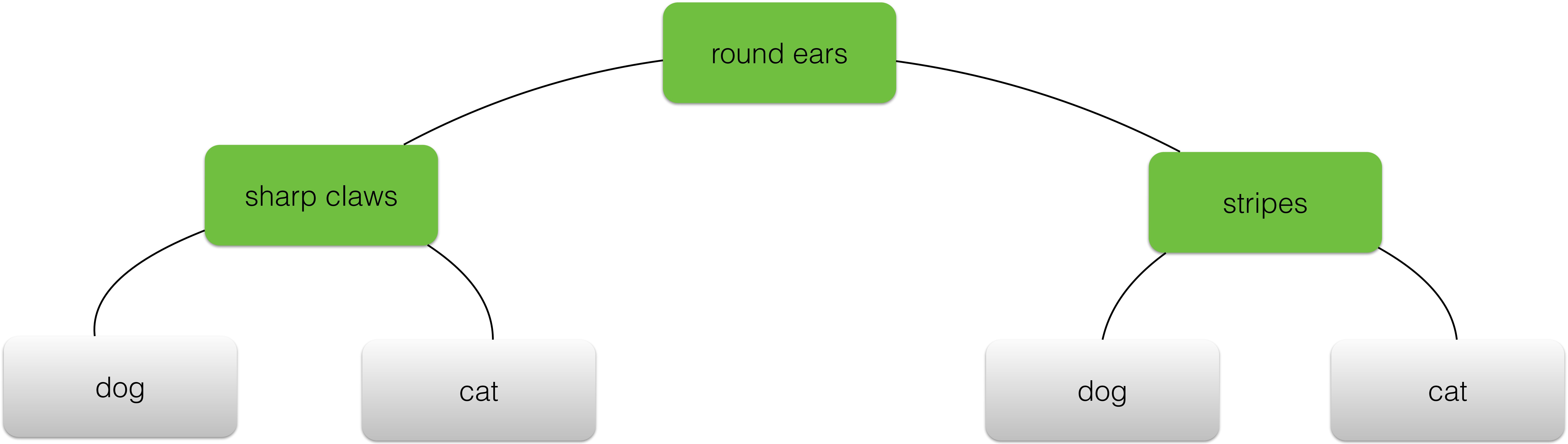
- A decision tree can achieve 100% training accuracy when each example is unique
- Limit depth of tree
- Strategy: train very deep tree
  - Adaptively prune

# Pruning with Validation Set




Validation accuracy: 0.4

# Pruning with Validation Set

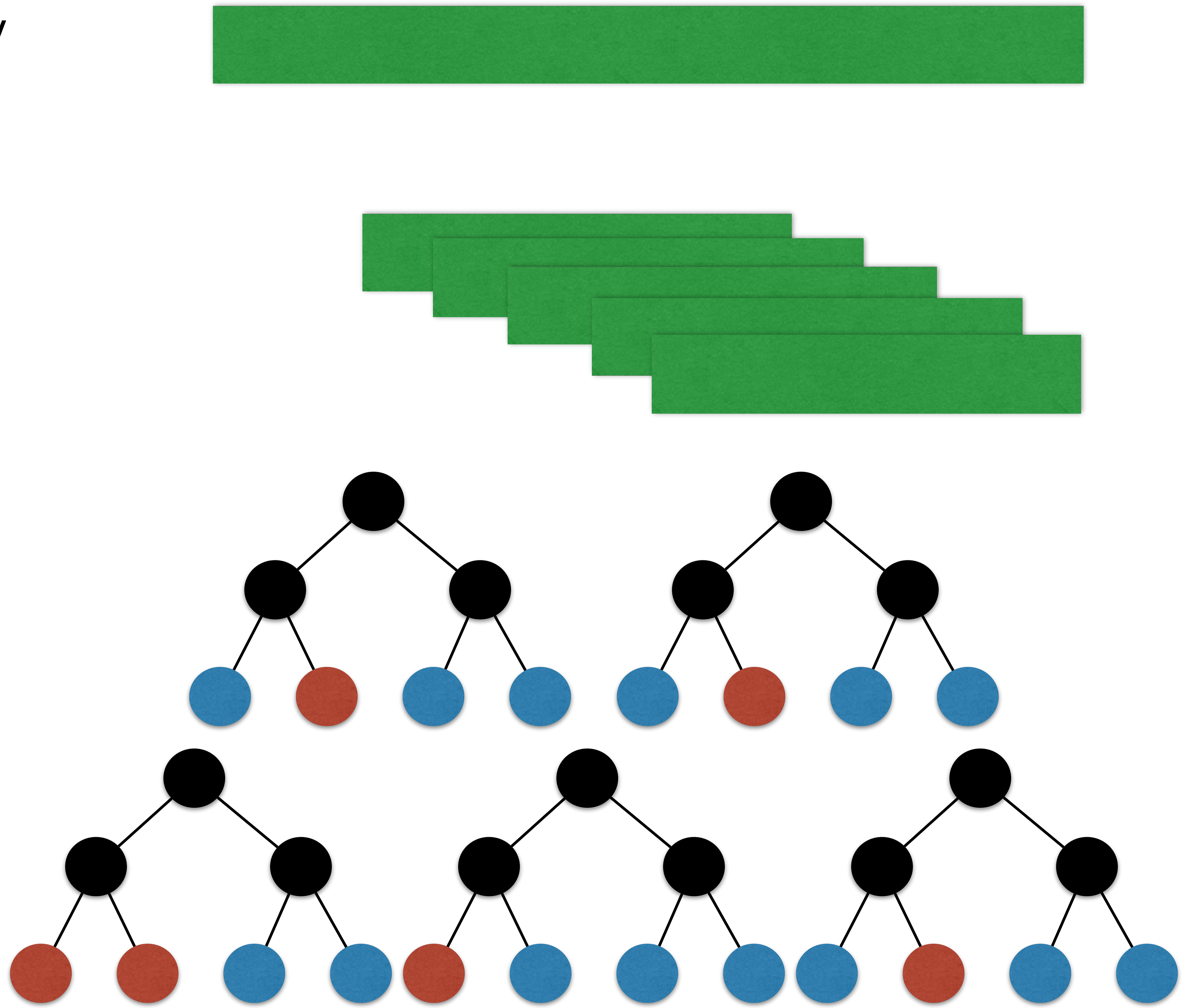



Validation accuracy: 0.4

new validation accuracy: 0.41

# Random Forests

- Use **bootstrap aggregation** to train many decision trees
  - Randomly subsample **n** examples
  - Train decision tree on subsample
  - Use average or majority vote among learned trees as prediction
- Also randomly subsample features
- Reduces variance without changing bias



# Summary

- Training decision trees
- Cost functions
  - Misclassification
  - Entropy and information gain
  - Gini index (expected error)
- Pruning
- Random forests (bagging)