we have used a value of the cut-off frequency between 10 and 20 MeV, which gives realistic dissipation and fluctuation for the fusion or fission mechanism[14, 15, 18]. Our study clearly points out that a proper treatment of memory requires to include higher order effects. (iv) Finally, in all cases, TCL4 could not be distinguished from the exact result. As we will see, the efficiency of TCL4 is similar for the inverted parabola.

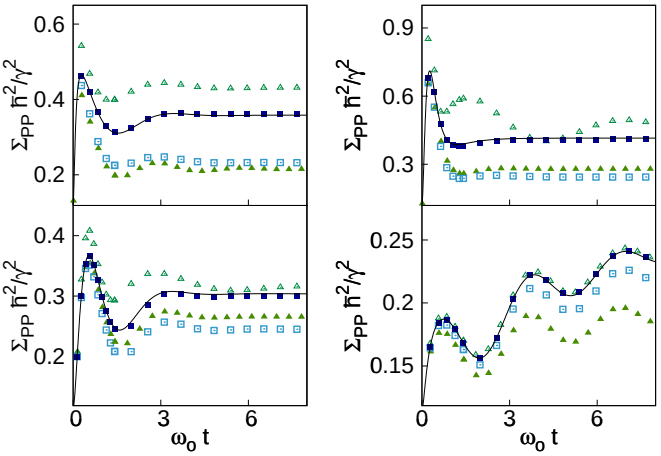

FIG. 3: (Color online) Evolution of Σ_{PP} for different approximations: NZ2 (open triangles), NZ4 (filled triangles), TCL2 (open squares) and TCL4 (filled squares). The exact evolution is displayed with solid line. In all cases, $\hbar\omega_0 = 14$ MeV, and $k_BT = \hbar\omega_0$ are used. The left side, corresponds to different cut-off frequencies: $\hbar\Omega = 20\hbar\omega_0$ (top) and $\hbar\Omega = 5\hbar\omega_0$ (bottom). In both cases, $\eta = 0, 5\hbar\omega_0$. In the right side, $\hbar\Omega = 10\hbar\omega_0$ and different coupling strengths are used: $\eta = \hbar\omega_0$ (top) and $\eta = 0.1\hbar\omega_0$ (bottom).

Since NZ method is not competitive, only the quantum Monte-Carlo and TCL methods are considered in the following application.

C. Quantum Monte-Carlo method applied to inverted oscillators

Several approaches have been recently developed to describe fusion and fission reactions [6, 14, 15, 17, 18, 53]. In these mechanisms, few collective degrees of freedom couple to a sea of internal excitations while passing an inverted barrier. At very low energy, both quantum and non-Markovian effects are expected to play a significant role. Most of the theory currently used start from quantum master equations deduced from TCL2. The quantum Monte-Carlo method offers a practical alternative which has similarities with path integrals theory. Path integrals are known to provide a possible framework to include dissipation while passing barriers (see for instance [54]). However, due to their complexity, only few applications have been made so far [2, 55]. We compare here the different approaches for inverted potential ($\varepsilon = -1$).

1. *Initial conditions, trajectories and mean evolution*

Initially, we consider a Gaussian density with quantum width $\sigma_{QQ}(0) = 0.16$ fm^2 and $\sigma_{PQ}(0) = 0$ MeV.fm/c and positioned on one side of the potential (here taken arbitrarily at $\langle Q(0)\rangle = Q_0 > 0$ while the barrier height is is located at 0 fm and is by convention taken as $V_B = 0$ MeV). The initial kinetic energy, denoted $E_K(0)$ is set by boosting the density with an initial momentum $\langle P(0)\rangle = P_0 < 0$.

Contrary to the classical theory of Brownian motion, the notion of trajectories is not so easy to tackle in the present Monte-Carlo framework. First, observables are complex. As mentioned in section III A, this difficulty can be overcomed by grouping trajectories by pairs which is equivalent to replace expectation of observables by their real parts. Second, it should be kept in mind that the present theory is a purely quantum theory where densities associated to wave-packets are evolved. Therefore, each trajectory should be interpreted in the statistical sense of quantum mechanics and contains many classical paths. Nevertheless, to visualize the trajectory we define the following energies:

$$E(t) = \frac{P(t)^2}{2m} - \frac{1}{2}m\omega_0^2 Q(t)^2 \tag{26}$$

where $Q(t)$ and $P(t)$ denote the real part of $\langle Q(t)\rangle$ and $\langle P(t)\rangle$ along the trajectory. An illustration of two trajectories, one passing the barrier and one reflected is shown in figure 4. As illustrated in the following, it is convenient to group trajectories according to the quantity ΔE defined by

$$\Delta E \ = \ E(0) - V_B \tag{27}$$

which is nothing but the difference between total initial energy and barrier high. Both trajectories shown in figure 4 correspond to $\Delta E = 0$ MeV.

It is tempting to group trajectories into those passing the barrier and those reflected by the potential to get information on the passing probability or passing time, however, it should be kept in mind that the present theory is fully quantal. Since each trajectories are associated with densities with quantum widths, both trajectories presented in Fig. 4 contribute to the transmission probability.

The accuracy of different methods is illustrated in figure 5 where evolutions of $\langle Q\rangle$, $\langle P\rangle$, Σ_{QQ} and Σ_{PP} are shown as a function of time. Values of parameters retained for this figure are typical values generally taken in the nuclear context[16]. In all cases, including TCL2, second moments are well reproduced. However, only TCL4 and the stochastic simulation provides a correct description of first moments. Calculations are shown here for $\Delta E = 0$ MeV. TCL2 provides a better and better approximation when ΔE increases while the disagreement increases below the barrier. This will be further illustrated below.